Phys 218 — Challenge Exam Formulae

**Trigonometry and Vectors:**

\[
\begin{align*}
\sin 30^\circ &= \cos 60^\circ = \frac{1}{2} \\
\sin 45^\circ &= \cos 45^\circ = \frac{1}{\sqrt{2}} \\
\sin 60^\circ &= \cos 30^\circ = \frac{\sqrt{3}}{2}
\end{align*}
\]

\[
\begin{align*}
h_{\text{adj}} &= h \cos \theta = h \sin \phi \\
h_{\text{opp}} &= h \sin \theta = h \cos \phi \quad \frac{h^2}{h_{\text{adj}}} = h_{\text{opp}}^2
\end{align*}
\]

Law of cosines: \( C^2 = A^2 + B^2 - 2AB \cos \gamma \)

Law of sines: \( \frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C} \)

\[
\begin{align*}
\vec{A} &= A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k} \\
\vec{B} &= B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}
\end{align*}
\]

\[
\vec{A} \cdot \vec{B} = A_1B_1 + A_2B_2 + A_3B_3 = AB \cos \theta = A_1B_1 = \vec{A} \cdot \vec{B}
\]

\[
\vec{A} \times \vec{B} = (A_2B_3 - A_3B_2) \hat{i} + (A_3B_1 - A_1B_3) \hat{j} + (A_1B_2 - A_2B_1) \hat{k}
\]

\[
A_{\text{B}} = AB \sin \theta = A_1B_1 (\text{direction via right-hand rule})
\]

**Kinematics:**

**translational**  \( \vec{v} = \frac{d\vec{r}}{dt} \)  \( \vec{a} = \frac{d\vec{v}}{dt} \)

**rotational**  \( \vec{\omega} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{r}}{dt^2} \)  \( \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \)

\[
\begin{align*}
\vec{v}(t) &= \vec{v}_0 + \int_0^t \vec{a}(t') dt' \\
\vec{v}(t) &= \vec{v}_0 + \int_0^t \vec{a}(t') dt' \\
\vec{v}(t) &= \vec{v}_0 + \vec{a}t \\
\omega(t) &= \omega_0 + \int_0^t \omega(t') dt'
\end{align*}
\]

Constant (linear/angular) acceleration only

\[
\begin{align*}
\vec{v}(t) &= \vec{v}_0 + \vec{a}t + \frac{1}{2} \vec{a}t^2 \\
\omega(t) &= \omega_0 + \int_0^t \omega(t') dt'
\end{align*}
\]

\[
\omega(t) = \omega_0 + \int_0^t \omega_0 dt + \omega(t) dt
\]

\[
\begin{align*}
\vec{v}^2 &= \vec{v}_{\text{B}}^2 \quad \text{y} = v_{B\text{y}}^2 + 2\alpha_x (x - x_0) \\
\omega^2 &= \omega_0^2 + 2\alpha(t - \theta_0)
\end{align*}
\]

\[
\begin{align*}
\vec{r}(t) &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \\
\vec{v}(t) &= \vec{v}_0 + \vec{a} t \\
\omega(t) &= \omega_0 + \int_0^t \omega(t') dt'
\end{align*}
\]

\[
\begin{align*}
\vec{F} &= \vec{F}_\text{L} + \vec{F}_\text{T} \\
\vec{F} &= \vec{F}_\text{L} + \vec{F}_\text{T} + \vec{F}_\text{applied}
\end{align*}
\]

\[
\begin{align*}
W &= \int \vec{F} \cdot d\vec{r} \quad \text{torque} \\
W &= \int \tau dt \\
P &= \frac{dW}{dt} \\
\vec{L} &= \sum \vec{r} \times \vec{p}
\end{align*}
\]

\[
\begin{align*}
\vec{p}_{\text{cm}} &= m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots \\
&= M \vec{v}_\text{cm} \\
\vec{J} &= \int \vec{r} \times d\vec{m} \\
\vec{F}_\text{ext} &= M \vec{a}_\text{cm} + \frac{d\vec{p}_{\text{cm}}}{dt} \\
\vec{F}_\text{int} &= 0
\end{align*}
\]

\[
\begin{align*}
\vec{F} &= -\vec{\nabla} U \\
F_x &= -dU(x)/dx \\
F_x &= -\frac{\partial U}{\partial x}
\end{align*}
\]

\[
\begin{align*}
W &= \Delta K \\
E_\text{tot, i} + W_\text{other} &= E_\text{tot, f}
\end{align*}
\]

\[
\begin{align*}
U &= -\int \vec{F} \cdot d\vec{r} \\
U_{\text{grav}} &= M g y_{\text{cm}} \quad U_{\text{class}} = \frac{1}{2} K \Delta x^2
\end{align*}
\]

\[
\begin{align*}
F_x &= -dU(x)/dx \\
\vec{F} &= -\vec{\nabla} U \\
\vec{F} &= -\frac{\partial U}{\partial x}
\end{align*}
\]

**Quadratic:**

\[
ax^2 + bx + c = 0 \quad \Rightarrow \quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Derivatives:**

\[
\frac{d}{dt} (at^n) = nat^{n-1}
\]

\[
\frac{d}{dt} \sin at = a \cos at \\
\frac{d}{dt} \cos at = -a \sin at
\]

**Integrals:**

\[
\begin{align*}
\int f(t) dt &= \frac{a}{n+1} (t^{n+1} - t_1^{n+1}) \\
\int f(t) dt &= \frac{a}{n+1} t_1^{n+1} + C
\end{align*}
\]

\[
\begin{align*}
\int f(t) dt &= \frac{a}{n+1} t_1^{n+1} + C \\
\int \cos at dt &= \frac{1}{n} \sin at
\end{align*}
\]

**Constants/Conversions:**

\[
\begin{align*}
g &= 9.80 \text{ m/s}^2 = 32.15 \text{ ft/s}^2 \quad \text{(Earth, sea level)} \\
&\approx 10 \text{ m/s}^2 \approx 33 \text{ ft/s}^2
\end{align*}
\]

\[
G = 6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \quad 1 \text{ mi} = 1609 \text{ m} \\
1 \text{ lb} = 4.448 \text{ N} \\
&\Rightarrow 0.454 \text{ kg} \quad 1 \text{ in} = 2.54 \text{ cm} \\
&1 \text{ rev} = 360^\circ = 2\pi \text{ radians}
\]

**Circular motion:**

\[
\begin{align*}
a_{\text{rad}} &= \frac{v^2}{R} \\
a_{\text{tan}} &= \frac{d[v]}{dt} = Ra
\end{align*}
\]

\[
T = \frac{2\pi R}{v} \\
s = R \theta \\
v_{\text{tan}} = R \omega
\]

**Relative velocity:**

\[
\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C} \\
\vec{v}_{A/B} = -\vec{v}_{B/A}
\]

**Forces:**

**Newton’s:**

\[
\sum \vec{F} = m \vec{a} \\
\vec{F}_{B \text{ on } A} = -\vec{F}_{A \text{ on } B}
\]

**Hooke’s:**

\[
F_x = -k \Delta x
\]

**friction:**

\[
|\vec{f}_s| \leq \mu_s |\vec{n}|, \quad |\vec{f}_k| = \mu_k |\vec{n}|
\]

**Centre-of-mass:**

\[
\vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots + m_n \vec{r}_n}{m_1 + m_2 + \ldots + m_n}
\]

(and similarly for \( \vec{v} \) and \( \vec{a} \))
Moments of inertia:

- Slender rod, axis through centre: \( I = \frac{1}{12}ML^2 \)
- Slender rod, axis through one end: \( I = \frac{1}{3}ML^2 \)
- Rectangular plate, axis through centre: \( I = \frac{1}{12}M(a^2 + b^2) \)
- Thin rectangular plate, axis along edge: \( I = \frac{1}{3}Ma^2 \)
- Hollow cylinder: \( I = \frac{1}{2}M(R_1^2 + R_2^2) \)
- Solid cylinder: \( I = \frac{1}{2}MR^2 \)
- Thin-walled hollow cylinder: \( I = MR^2 \)
- Solid sphere: \( I = \frac{2}{3}MR^2 \)
- Thin-walled hollow sphere: \( I = \frac{2}{3}MR^2 \)

\( \Rightarrow \) For a point-like particle of mass \( M \) a distance \( R \) from the axis of rotation: \( I = MR^2 \)

\( \Rightarrow \) Parallel axis theorem: \( I_p = I_{cm} + Md^2 \)